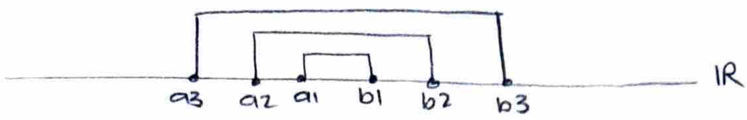
# Exercise 1.5.16

a.




First prove that if $A_i$ is a closed interval with endpoints $a_i \leq b_i$, that for all $i, j \in I$ have $a_i \leq b_j$:

We do this in two cases:

case1:

if $A_i \subseteq A_j$, then $a_i \geq a_j \cap b_i \leq b_j$

since $a_i \leq b_i \Rightarrow a_i \leq b_j$

cas2:

if $A_j \subseteq A_i$, then $a_i \leq a_j \cap b_i \geq b_j$

since $a_j \leq b_j \Rightarrow a_i \leq b_j$

Thus, we have praven what we wanted to.


Thus, the set $\{a_i \in \mathbb{R} \mid i \in I\}$ has all $b_j$ as it's upper bounds, since $a_i \leq b_j$.

Because $\mathbb{R}$ is a complete total ordered field, $\{a_i \in \mathbb{R} \mid i \in I\}$ must have a supremum, we will call c

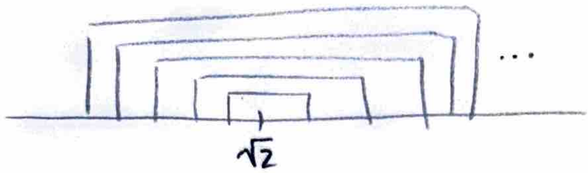
$c \geq a_i$, and

$c \leq b_j$ because it is the least upper bound.

Thus $a_i \leq c \leq b_j$, and so all $A_i$ contains $c$.

3

b.




$\in \mathbb{Q}$

$A = \{$ interval $A_i$ with bandaryes $a_i, b_i \mid a_i^2 \leq x$ , $b_i^2 \geq x \}$

where $x = 2$

This family cannot ~~havecotn~~ ~~of~~ a non-emty ~~inter~~ intersechion